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Predictive Modelling of Cutting Force in a Straight Turning Operation

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Abstract: *In this research, a second order polynomial ridge regression model was developed for the cutting force in the straight turning of "AISI 1005" steel wherein cutting force was regressed on cutting speed, feed rate and depth of cut. Design of experiments was used to reduce the number of the experimental runs and provide near-optimum experimental condition. The result from the predictive model was compared to the experimental result. The error – in terms of the mean absolute error (MAE) and mean squared error (MSE) - from the ridge regression model was within acceptable bounds since the value of the prediction was very close to the experimental result using a validation data set that is completely different from the development data set.*

Keywords: *Cutting Force, Design of Experiments, Optimum, Mean Squared Error*

INTRODUCTION

In all machining operations including turning, energy conversion takes place where the cutting energy is transformed into heat which in turn translates to temperature rise in the cutting zone. Heat is generated from work done in shearing in the shear zone; friction between the tool and the machined surface; and friction at the tool-chip interface. It is important that this temperature increase be effectively managed as excessive temperature can have adverse effect on the cutting tool, workpiece or both (Kalpakjian and Schmid, 1999). Under 'dry machining' and 'high speed machining' conditions, excessive cutting force and could result in:

1. reduction in the strength, hardness and wear resistance of the tool
2. cutting tool softening and subsequent plastic deformation which in effect may alter the shape of the tool

Accordingly, the dimensional accuracy of the workpiece could be reduced; thermal and metallurgical damages could be passed on to the finished workpiece thereby affecting its in-service properties. Cutting force needs to be properly controlled during turning process. Excessive cutting force could cause deflection especially in long and/or slender workpieces. This results in elastic deformation which impacts the dimensional accuracy of the finished workpiece. Low cutting force on the other hand may not be enough to overcome the shear strength of the workpiece material to produce the desired cutting action. Either way the job is not done or is done improperly. The controllable independent variables - cutting speed, feed, and depth of cut – tends to influence the cutting force, temperature rise and tool life, type of chip formed and surface finish and integrity. The interaction of the input and output variables in metal machining is generally very complex. Some reasons for this complexity include but not limited to:

i. The large number of variables at play
ii. Some of the variables are simply outside the control of the machinist or experimenter
iii. The effect of one independent variable on a depended variable may be confounded by that of another independent variable on the same dependent variable.

The mechanical properties (strength, hardness, stiffness and wear-resistance) of the cutting tool are affected by the excessive heat at the tool-workpiece interface to the extent that the tool could become soft and experience plastic deformation. This in turn reduces the dimensional accuracy of the tool and eventually leads to tool failure. So, tool-chip temperature imparts greatly on the rate of tool wear (Usui *et al.*, 1978). According to Endrino *et al.* (2006), high temperatures at the tool–chip interface increases diffusion and chemical wear of both the tool and the workpiece. Using high cutting speed and unfavorable condition during machining results in very high temperature (Chattopadhyay and Chattopadhyay, 1982; Singh *et al.*, 1997). Plastic deformation of the cutting edge and rapid wear occurs at elevated temperature and pressure leading to dimensional inaccuracy, increased cutting forces and premature tool failure (List *et al.*, 2005). Higher power is consumed and more heat is generated as the cutting force is increased. The resultant tool wear and increased heat can induce change and/or damage in the thermal and metallurgical properties of machined surface. Also, Bouzid *et al.* (2004) reported that flank wear causes an increase in the cutting force and the interfacial temperature, reduction in dimensional accuracy in the finished work pieces and vibration which makes the cutting operation less efficient. High feed rate/speed inherently generates high cutting zone temperature. Sultana *et al.* (2009) reported that uncoated carbide insert creates more cutting temperature than coated insert when turning different steels.

Production economy in turning operation was investigated by Khan and Ahmed (2008) and they found that turning difficult-to-cut materials (like Stainless steel, Titanium, Inconel etc.) using existing conventional techniques is uneconomical as the turning process results in high tool wear; takes longer time and requires high cutting force. While turning these materials, the heat generated was very high due to strong adhesion between the tool and workpiece. This is as a result of their low thermal conductivity, high work hardening rate, high viscosity, high reactivity, tendency to form built-up-edge (BUE) at tool edge compared to other alloy steels. The hardness, plastic modulus and the fracture toughness of the tool decline with increase in cutting temperature, which accelerates tool wear rate was observed by Reed and Clark (1983). Moreover, thermal stresses in the tool increase with the temperature resulting in more cracks in the tool and premature failure of the tool. The amount of energy dissipated through the rake face of the tool raises the temperature at the flanks of the tool (Wu and Matsumoto, 1990). Ezugwu and Tang (1995) showed that important parameters including the choice of tool and coating materials, tool geometry, machining method, cutting speed, feed rate, depth of cut, lubrication, must be controlled in order to achieve adequate tool lives and surface integrity of the machined surface. In machining a given material, the tool life is governed mainly by the tool material which also influences cutting forces and temperature as well as accuracy and finish of the machined surface. A common issue with analysing machining operations is the lack of or absence of empirical models or equations to aid such analysis. Even when such equations exist, they are too complex and analysing them becomes cumbersome and sometimes near-impossible. This research sets out to establish a relatively less complex but representative empirical model relating cutting force to depth of cut, feed and cutting speed in order to aid the estimation and analysis of cutting force in a straight turning operation.

MATERIAL AND METHOD

2.1 Materials

The workpiece material is "**AISI 1005**" steel. This was chosen because it is a relatively common engineering material and it is economical. Another reason for its choice is the dissimilarity in mechanical properties with the cutting tool.

Equipment

i. **Machine Tool Dynamometer**. This was used to measure the machining force exerted by the machine tool on the workpiece during machining operation.

ii. **Cemented Carbide Cutting Tool**. Compared to the workpiece material, this cutting tool material has much higher resistance to heat and wear and thus ensures that effects of the machining operation on the cutting tool will be minimal and almost entirely felt by the workpiece. This is consistent with aim which is to model the cutting force on the workpiece as functions of feed, depth of cut and cutting speed.

2.2 Method

Thirty (30) cylindrical workpieces of length L = 50mm were straight turned from an initial diameter D_0 = 20mm to some final diameter D_f, with a lathe machine. The input variables, their units and ranges are as shown in Table 1. About 83% of the dataset was randomly chosen for training the model while the remaining 17% was used for validation. This resulted in 25 data points for training and 5 data points for validation.

Table-1 Input Variables and their ranges

Variable	Unit	Range
Feed	mm/rev	0.05 - 0.20
Depth of cut	mm	0.5 - 1.50
Cutting Speed	mm/min	90 - 135

The output variable, cutting force was measured with the dynamometer – attached to the cutting tool - and the cutting speed was gotten from the machine spindle speed, N using relation 1

$$v = \frac{\pi N(D_0 + D_f)}{2} \quad (1)$$

A second order polynomial model of the form was fitted to the data set. The choice of whether the model will be linear or ridge was determined by the presence of collinearity in the collected data set. Collinearity was determined by computing the eigenvalues of matrix X'X and thereafter evaluating the condition number given by

$$condition\ number, k(X) = \left[\frac{\lambda_{max}(X'X)}{\lambda_{min}(X'X)}\right]^{\frac{1}{2}} \quad (2)$$

$$\text{Where } X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{13}{}^2 \\ x_{21} & x_{22} & \cdots & x_{23}{}^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{n3}{}^2 \end{bmatrix}, n = 25$$

A large value of condition number would indicate that there is collinearity among the variables and consequently, ridge regression would be a better option to model the system. Having established collinearity, the ridge regression;

$$\beta = (X'X + kI)^{-1}X'y \quad (3)$$

Was fitted to the training set, with k being the shrinkage parameter that ensures that X'X is always invertible. Equation (3) was performed on the scaled elements of X using the relation;

$$z_{ij} = \frac{x_{ij} - \bar{x}_j}{\sigma_{xj}^2} \quad (4)$$

Where x_{ij} is the ith element of the jth row of X, $\bar{x}_j$ is the mean of the jth column of X and σ_{xj}^2 is the variance of jth column of X

Equation (3) becomes;

$$\beta = (Z'Z + kI)^{-1}Z'y \quad (5)$$

To choose an appropriate value of ridge parameter, a ridge plot of 500 possible values of k made against the standardized ridge coefficients to determine a value of k at which the system is stable. Having estimated the model parameters, the validation data set was used to ascertain the predictive ability of the model.

RESULTS AND DISCUSSION

3.1 Results

The measured cutting forces from the experiment are as shown in Table 2.

Table -2 Cutting Force Measurements

Depth of Cut (x_1)	Feed (x_2)	Cutting Speed (x_3)	Cutting Force (y)
0.66	0.18	108.777	100.84
1.29	0.06	92.2344	68.54
0.81	0.11	130.6222	75.37
1.03	0.09	132.5154	77.31
0.67	0.17	112.0889	95.57
1.10	0.11	112.0164	106.76
0.76	0.19	105.1974	120.25
1.15	0.08	130.5024	75.35
1.19	0.09	106.6161	90.00
1.25	0.07	95.0041	75.69
0.95	0.07	125.1113	56.52
0.58	0.18	107.5382	88.95
0.73	0.14	100.8761	84.35
1.41	0.13	108.176	158.16
0.65	0.07	94.3405	39.51
1.33	0.18	95.9388	199.32
1.04	0.14	132.3923	125.67
1.49	0.10	133.0261	129.65

0.58	0.13	115.8844	62.02
0.94	0.11	92.6901	87.82
0.61	0.06	100.5651	31.46
1.46	0.09	105.8921	106.19
0.50	0.07	126.9537	29.19
1.27	0.08	90.6932	83.56
1.32	0.09	91.9361	95.68

The mean, standard deviation and other summary statistics for the cutting temperature, depth of cut, feed and cutting speed are shown in Table 3.

Table-3 Summary Statistics for Variables

	Depth of Cut	Feed	Cutting Speed	Cutting Force
Count	25	25	25	25
Mean	0.991752	0.111388	109.9035	90.54968
STD	0.316255	0.041631	14.66209	37.629902
Min	0.5046	0.0614	90.6932	29.193995
25%	0.6656	0.0773	95.9388	75.349199
50%	1.0285	0.1026	107.5382	87.822283
75%	1.2749	0.137	125.1113	106.187115
Max	1.4961	0.1866	133.0261	199.321305

A. Determining Collinearity

The data matrix, X with single, two-factor interaction and quadratic effects.

$$X = [x_1 \ x_2 \ x_3 \ \ x_1 x_2 \ \ x_1 x_3 \ x_2 x_3 \ x_1{}^2 \ x_2{}^2 \ x_3{}^2]$$

Computing the eigenvalues of X'X matrix gives

$$\lambda_{1,2,\ldots,9} = \begin{bmatrix} 4.04E+09 \\ 35880.63 \\ 4196.187 \\ 432.186 \\ 0.251455 \\ 0.020488 \\ 0.004402 \\ 2.79E-05 \\ 0.000255 \end{bmatrix}$$

$$condition\ number, k(X'X) = \left[\frac{4.04E+09}{2.79E-05}\right]^{0.5}$$

$$= 1.203E+07$$

The very large value of the condition number indicates multicollinearity. Consequently, simple linear regression would not be appropriate for modeling the system. Ridge regression is a better candidate for modeling this particular system.

B. Scaling Variables

Applying equation 4 to the data set gives the scaled variables shown in Table 4.

Table-4 Scaled Variables

x_1	x_2	x_3	$x_1 x_2$	$x_1 x_3$	$x_2 x_3$	x_1^2	x_2^2	x_3^2	y
-1.042	1.655	-0.077	0.274	-0.993	1.568	-1.014	1.780	-0.137	0.274
0.957	-1.170	-1.205	-0.585	0.297	-1.377	0.942	-0.979	-1.144	-0.585
-0.571	-0.033	1.413	-0.403	-0.069	0.451	-0.667	-0.190	1.446	-0.403
0.116	-0.538	1.542	-0.352	0.757	-0.097	-0.034	-0.594	1.597	-0.352
-1.031	1.408	0.149	0.133	-0.923	1.449	-1.007	1.431	0.084	0.133
0.349	0.080	0.144	0.431	0.407	0.128	0.213	-0.088	0.079	0.431
-0.723	1.807	-0.321	0.789	-0.769	1.572	-0.787	2.003	-0.369	0.789
0.513	-0.819	1.405	-0.404	1.148	-0.460	0.399	-0.781	1.437	-0.404
0.624	-0.523	-0.224	-0.015	0.498	-0.574	0.529	-0.583	-0.278	-0.015
0.811	-0.951	-1.016	-0.395	0.275	-1.156	0.757	-0.861	-0.987	-0.395
-0.130	-0.985	1.037	-0.904	0.284	-0.733	-0.279	-0.880	1.020	-0.904
-1.290	1.658	-0.161	-0.042	-1.245	1.523	-1.168	1.783	-0.218	-0.042
-0.831	0.615	-0.616	-0.165	-0.952	0.334	-0.867	0.453	-0.639	-0.165
1.333	0.507	-0.118	1.797	1.209	0.444	1.452	0.336	-0.177	1.797
-1.073	-0.953	-1.061	-1.356	-1.279	-1.168	-1.034	-0.862	-1.025	-1.356
1.056	1.600	-0.952	2.891	0.510	1.028	1.073	1.700	-0.933	2.891
0.147	0.767	1.534	0.933	0.789	1.432	-0.002	0.624	1.587	0.933
1.595	-0.211	1.577	1.039	2.466	0.298	1.833	-0.342	1.638	1.039
-1.308	0.375	0.408	-0.758	-1.130	0.526	-1.179	0.199	0.346	-0.758
-0.155	-0.026	-1.174	-0.072	-0.575	-0.431	-0.302	-0.184	-1.118	-0.072
-1.218	-1.201	-0.637	-1.570	-1.294	-1.294	-1.125	-0.994	-0.658	-1.570
1.487	-0.610	-0.274	0.416	1.262	-0.669	1.673	-0.644	-0.325	0.416
-1.540	-1.030	1.163	-1.631	-1.210	-0.757	-1.305	-0.905	1.160	-1.631
0.895	-0.812	-1.310	-0.186	0.194	-1.110	0.863	-0.777	-1.229	-0.186
1.029	-0.610	-1.225	0.136	0.344	-0.925	1.037	-0.644	-1.161	0.136

C. Choosing Ridge Parameter, k

500 values of the ridge parameter from 0 to 0.005 with interval of 0.00005 were generated and the corresponding ridge estimates $\boldsymbol{\beta}_R$ computed. The plot of the ridge parameters against the standardized coefficients is shown in Fig. 1.

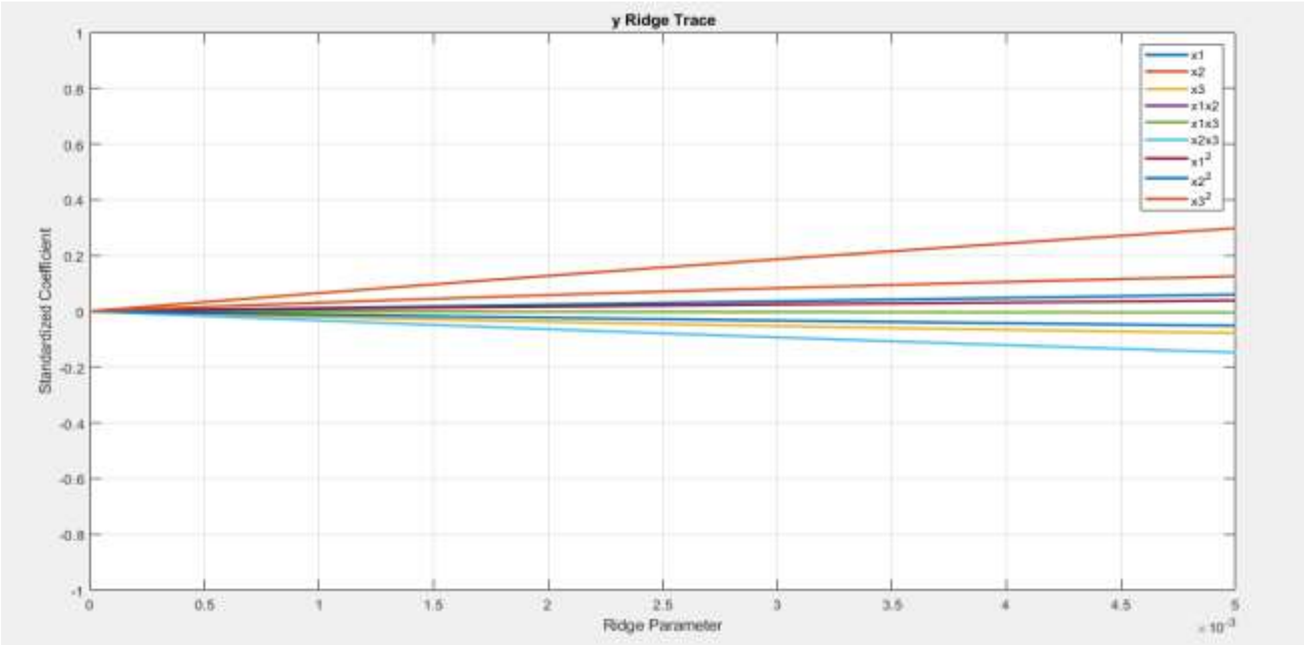

Fig. 1 Ridge trace of Cutting Force

The system is stable when k is 0. As k increases, the system's stability reduces. A value of k = 2.5x10^{-5} is small enough not to significantly increases the bias of the system and just large enough to maintain low variance and hopefully ensure it generalises well to external data sets.

D. Ridge Regression

$$\beta = (X'X + 0.000025 I_9)^{-1} X' y_1 \qquad (6)$$

Solving the above gives,

$$\beta = \begin{bmatrix} 2.9374E-04 \\ 0.0017 \\ -5.4066E-04 \\ 37.6292 \\ -2.4332E-05 \\ -8.4735E-04 \\ 2.1661E-04 \\ -3.1150E-04 \\ 8.1018E-04 \end{bmatrix} \qquad (7)$$

Converting the above to the origin variables scale we have,

$$\bar{X} = [\,0.9918 \;\; 0.1114 \;\; 109.904 \;\; 0.1072 \;\; 108.4822 \;\; 12.2496 \;\; 1.0796 \;\; 0.0141 \;\; 12285]$$
$$s = [\,0.3163 \;\; 0.0416 \;\; 14.6621 \;\; 0.0446 \;\; 36.7078 \;\; 4.6959 \;\; 0.6323 \;\; 0.0104 \;\; 3302.6710]$$
$$\bar{y} = 90.54968$$

$$\beta./s = \begin{bmatrix} 0.0009288 \\ 0.004 \\ -0.0000336875 \\ 844.5940 \\ -0.00000066269 \\ -0.00018045 \\ 0.00034259 \\ -0.0301 \\ 0.00000024531 \end{bmatrix} \qquad (8)$$

$\bar{X}(\beta./s) = 90.5501$
$(\bar{y} - \bar{X}(\beta./s) = 90.54968 - 90.5501 = -0.000403995$

$$\beta_{tr} = \begin{bmatrix} -4.0400E-04 \\ 9.2880E-04 \\ 0.0400 \\ -3.6875E-05 \\ 844.5940 \\ -6.6290E-07 \\ -1.8045E-04 \\ 3.4259E-04 \\ -0.0301 \\ 2.4531E-07 \end{bmatrix} \quad (9)$$

The regression equations become,

$$y = X \begin{bmatrix} -4.0400E-04 \\ 9.2880E-04 \\ 0.0400 \\ -3.6875E-05 \\ 844.5940 \\ -6.6290E-07 \\ -1.8045E-04 \\ 3.4259E-04 \\ -0.0301 \\ 2.4531E-07 \end{bmatrix} \quad (10)$$

3.2 Discussion

At least two conditions forces collinearity on the system: physical constraints – due to factor settings or ranges; the model specification – from the interactions and second order effects. A third factor has to do with the narrow subspace from which the data was collected. The above factors result in inaccurate regression coefficient estimates, inflates the standard error of the regression coefficients and degrades the prediction capacity of the model among other issues. The ridge regression model aims to minimize these concerns and stabilize the model for projection outside the design space. Using the validation data set of Table 4.4, matrix X is generated thus,

$X = [x_1\ x_2\ x_3\ x_1x_2\ x_1x_3\ x_2x_3\ {x_1}^2\ {x_2}^2\ {x_3}^2]$
Applying 5 to X gives the predicted values in the "y predicted" column of Table 5.

Table-5 Validation Data

X_1	X_2	X_3	y
0.87	0.17	135.56	127.33
1.24	0.20	135.31	208.96
0.80	0.13	135.37	90.77
0.95	0.12	135.80	99.30
0.59	0.10	135.66	49.00

Table-6 Model Performance

Y	Y predicted	Error, e	e^2	|e|
127.33	127.3272	0.0028	0.00000804	0.002836
208.96	208.9634	-0.0034	0.00001156	0.003401
90.77	90.76608	0.0039	0.00001537	0.003920

99.30	99.30215	-0.0021	0.00000460	0.002146
49.00	48.99731	0.0027	0.00000724	0.002690

The value of the ridge parameter k = 0.000025 was sufficient to stabilize the model against variance from an external data set and small enough to minimize the bias on the training data set.

$$Mean\ Squared\ Error, mse_1 = \frac{\sum e_1{}^2}{n} = \frac{0.00004681}{5} = 0.00000936$$

$$Mean\ Absolute\ error\ mae_1 = \frac{\sum |e_1|}{n} = \frac{0.014992}{5} = 0.002998$$

This is evident from the small magnitude in error, |e| on model validation from Table 6. The model specification and parameter tuning have significantly minimized the inherent multicollinearity in the data set and has improved the predictability of the model.

CONCLUSION

The carefully chosen ridge parameter values for the regression model gave satisfactory results in the form of low bias when the models were applied to the experimental data sets. This is evident from the performance measure values from the model which returned a mean absolute error value of 0.002998 and a mean squared error value of 0.00000936. This confirms the low variance of the ridge regression model and its predictive ability in the presence of external data.

CONFLICT OF INTEREST

I hereby state that no conflict of interest will arise in any form the publishing of this study.

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