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Yield Criterion Analysis of Thin Rectangular Plates

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Abstract: *Structural failures occur mostly due to elements that have been stressed beyond their ultimate strength and deformed beyond their allowable deflection as a result of applied loads. Plate is one of these structures experiencing these failures and leading to economic damages, plate analysis is complex unlike other structures and the structural engineer's aim is to design safe and economic structures. Therefore, this study is aimed at formulating a new general yield criterion equation or allowable stress equation and specific yield criteria equations or allowable stress equations for various thin rectangular plates using polynomial displacement shape functions. The strains of an isotropic two-dimensional element are substituted into the strain energy per unit volume expression and evaluated to obtain the general yield criterion or allowable stress equation in terms of n-value expressions. The displacement shape functions of each plate type are evaluated and substituted into the general allowable stress equation to obtain the specific allowable stress equation for each plate considered. A numerical application was carried out, and th e results indicated that for a safe design, the stress factor of safety should be 1.10. This has proven to be useful for easy analysis of thin rectangular plates which will save economic damages due to plate failures. The new equation will open a new dimension for yield analysis that will be beneficial to structural engineers and the Aerospace and Shipbuilding industries.*

Keywords: *Yield Criterion, Maximum Strain Energy, Allowable Stress, Stress Factor, Thin Rectangular Plates*

INTRODUCTION

It is of great importance in stress analysis, to consider the combination of stresses and strain that initiate yield in a material because the initiation of yield is most times related to the ultimate failure of the structure or material (Ross, 1987). The yield criterion is a hypothesis defining the limit of elasticity of material under any possible combination of stresses (Chakraberty, 2006; Singh, 2009; Benhan, and Warnock, 1980). The theories of yield are related to the triaxial principal stress system, such that, $\sigma_1 > \sigma_2 > \sigma_3$. Where σ_1 and σ_3 are the maximum and minimum principal stress respectively and σ_2 is the minimax principal stress. According to Ross (1987), the choice of a triaxial principal stress system to investigate yield criteria is that such a system describes the complete stress situation at a point, without involving the complexities caused by shear stresses on six planes, which would have resulted if a different 3-D coordinate system was used. A yield criterion is expressed generally as (Lee, 1977).

$$f(\sigma_{m,n}, c) = 0. \quad (1)$$

where $\sigma_{m,n}$ stress components, and 'c' are parameters like plastic strains. In his work, Lee (1977) assumed that the plastic behavior of the material is independent of time and temperature, that is, 'c' is zero, bringing Eq.1 to an alternative form as

$$f(\sigma_{m,n}) = k. \quad (2)$$

where k represents some yielding parameter, which is constant for the case of initial yielding. While f is the yield function that represents a hypersurface to bound all the accessible states which can be achieved in actual material elements by some program of stressing (Martin, 1975). The hypersurface may be projected into the stress space known as the yield surface by temporarily fixing 'c' as a constant at an instantaneous time. There are various theories of elastic failure as stated by Ross (1987). Some of these are; the maximum principal stress theory by Rankine, maximum principal strain theory by St. Venant, total strain energy theory by Beltrami and Haigh, maximum shear stress theory by Tresca, maximum shear strain (distortion) energy theory, and Octahedral shear stress theory both by Hencky and Von Mises (or Von Mises). However, the Tresca and Von Mises yield criteria are mostly used for metallic material (Moy, 1981; Ross, 1987; Lee, 1977; Save *et al.*, 1997) because, they considered the fact that shear controls yield, and they gave an accurate prediction of the onset of yield in ductile materials. Some other works in yield criteria analysis are those of (Christensen, 2006a; Christensen, 2006b; Chandrasekaran; Kuhl *et al.* 2006b; Lubliner). The present work is aimed at formulating new general yield criterion mathematical models in terms of the allowable stress for thin isotropic rectangular plates based on maximum strain energy. And using the polynomial displacement shape profile to formulate specific mathematical models for various plate types. This will simplify yield criteria analysis and guide yield conditions to prevent the failure of thin plate structures.

MATERIALS AND METHODS

A. Formulation of Yield Criterion Equation

This formulation of the yield criterion equation will be based on the maximum strain energy theory of yield.

B. Maximum Strain Energy Theory Approach

The strain energy per unit volume 'U' is given by

$$U = \frac{1}{2}\sigma\varepsilon. \quad (3)$$

Where σ is stress and ε is strain.

Expanding Eq.3 for a two-dimensional plate structure (x-y plane), we have

$$U = \frac{1}{2}\left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy}\right). \quad (4)$$

But strains for isotropic plates are given as;

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \upsilon\sigma_y\right). \quad (5a)$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_y - \upsilon\sigma_x\right). \quad (5b)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2(1+\upsilon)}{E}\tau_{xy}. \quad (5c)$$

where $G = \frac{E}{2(1+\upsilon)}$. (6)

E is the Young modulus of elasticity, τ_{xy} is shear stress, γ_{xy} is shear strain, and υ is the Poisson ratio.

Substituting Eq.5 in Eq.4 yields

$$U = \frac{1}{2E}\left[\sigma_x\left(\sigma_x - \upsilon\sigma_y\right) + \sigma_y\left(\sigma_y - \upsilon\sigma_x\right) + 2(1+\upsilon)\,\tau_{xy}{}^2\right]. \tag{7a}$$

Opening up internal bracket yields

$$U = \frac{1}{2E}\left[\sigma_x{}^2 - \upsilon\sigma_x\sigma_y + \sigma_y{}^2 - \upsilon\sigma_x\sigma_y + 2\tau_{xy}{}^2 + 2\upsilon\,\tau_{xy}{}^2\right]. \tag{7b}$$

Collecting like terms yields

$$U = \frac{1}{2E}\left[\sigma_x{}^2 + 2\tau_{xy}{}^2 + \sigma_y{}^2 - 2\upsilon\sigma_x\sigma_y + 2\upsilon\,\tau_{xy}{}^2\right]. \tag{7c}$$

Rearranging yields

$$U = \frac{1}{2E}\left[\sigma_x{}^2 + 2\tau_{xy}{}^2 + \sigma_y{}^2 + 2\upsilon(\tau_{xy}{}^2 - \sigma_x\sigma_y\right]. \tag{7d}$$

Therefore, at yield, $U = U_0$, hence

$$U_0 = \frac{1}{2E}\left[\sigma_x{}^2 + 2\tau_{xy}{}^2 + \sigma_y{}^2 + 2\upsilon(\tau_{xy}{}^2 - \sigma_x\sigma_y\right] \le U_{0max}. \tag{8}$$

Let $\sigma_x = f_y$. (9)

Let $\sigma_y = \tau_{xy} = 0$. (10)

where f_y is the yield stress of a material

Thus, Eq.8 reduces to

$$U_0 = \frac{1}{2E}\left[f_y{}^2\right] \le U_{0max}. \tag{11}$$

$$U_{0max} \ge \frac{1}{2E}\left[f_y{}^2\right]. \tag{12}$$

Substituting Eq.11 into Eq.8 yields

$$U_0 = \frac{1}{2E}\left[\sigma_x{}^2 + 2\tau_{xy}{}^2 + \sigma_y{}^2 + 2\upsilon(\tau_{xy}{}^2 - \sigma_x\sigma_y\right] \le \frac{1}{2E}\left[f_y{}^2\right]. \tag{13a}$$

That is

$$\sigma_x{}^2 + 2\tau_{xy}{}^2 + \sigma_y{}^2 + 2\upsilon\left(\tau_{xy}{}^2 - \sigma_x\sigma_y\right) \le f_y{}^2. \tag{13b}$$

From Eq.13b we have

$$\sigma_x{}^2\left[1 + \frac{2\tau_{xy}{}^2}{\sigma_x{}^2} + \frac{\sigma_y{}^2}{\sigma_x{}^2} + 2\upsilon\left(\frac{\tau_{xy}{}^2}{\sigma_x{}^2} - \frac{\sigma_x\sigma_y}{\sigma_x{}^2}\right)\right] \le f_y{}^2. \tag{14}$$

But stresses in a two-dimensional plane are

$$\sigma_x = -\frac{EZ}{(1-\upsilon^2)}\left(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}\right). \tag{15}$$

$$\sigma_y = -\frac{EZ}{(1-\upsilon^2)}\left(\upsilon\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right). \tag{16}$$

$$\tau_{xy} = -\frac{EZ(1-\upsilon)}{2(1-\upsilon^2)}\left(\frac{\partial^2 w}{\partial x\,\partial y}\right). \tag{17}$$

Where Z is the plate depth.

The displacement shape function, w, is given as

$$w = Ah \tag{18}$$

where A is the amplitude of deflection and h is the shape profile of the plate based on the plate type. If we take the aspect ratio of the plate to be;

$$Ƨ = \frac{b}{a} \tag{19}$$

In non-dimensional parameters,

$$x = aR,\quad y = bQ,\qquad z = St,\qquad 0 \le R \le 1, 0 \le Q \le 1, S = 0.5 \tag{20}$$

Substitute Eq. 18, Eq.19, and Eq.20 in Eq. 15, Eq.16, and Eq.17 yield equation s Eq. 21, Eq. 22 and Eq. 23 respectively.

$$\sigma_x = -\frac{EAZ}{(1-\upsilon^2)a^2}\left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right) \tag{21}$$

$$\sigma_y = -\frac{EAZ}{(1-\upsilon^2)a^2}\left(\upsilon\frac{\partial^2 h}{a^2\,\partial R^2} + \frac{1}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right) \quad (22)$$

$$\tau_{xy} = -\frac{EAZ(1-\upsilon)}{2a^2(1-\upsilon^2)Ƨ}\left(\frac{\partial^2 h}{\partial R\,\partial Q}\right) \quad (23)$$

$$\text{Let } m_1 = \frac{\sigma_y}{\sigma_x} \Rightarrow \sigma_y = m_1\sigma_x. \quad (24)$$

$$m_2 = \frac{\tau_{xy}}{\sigma_x} \Rightarrow \tau_{xy} = m_2\sigma_x. \quad (25)$$

Substitute Eq.24 to Eq.25 into Eq.14 yields

$$\sigma_x{}^2[1 + 2m_2{}^2 + m_1{}^2 + 2\upsilon(m_2{}^2 - m_1)] \le f_y{}^2. \quad (26)$$

$$\sigma_x{}^2[1 + m_1{}^2 - 2vm_1 + 2(1+\upsilon)m_2{}^2] \le f_y{}^2. \quad (27)$$

$$\sigma_x{}^2 \le \frac{f_y{}^2}{[1 + m_1{}^2 - 2vm_1 + 2(1+\upsilon)m_2{}^2]}. \quad (28)$$

$$\sigma_x \le \frac{f_y}{\sqrt{[1 + m_1{}^2 - 2vm_1 + 2(1+\upsilon)m_2{}^2]}}. \quad (29)$$

Eq.29 can be written as Eq.30

$$\sigma_x \le \frac{f_y}{F}. \quad (30)$$

Where F is the stress factor of safety, given by Eq.31

$$F = \sqrt{[1 + m_1{}^2 - 2vm_1 + 2(1+\upsilon)m_2{}^2]} = \sqrt{M}. \quad (31)$$

And

$$M = [1 + m_1{}^2 - 2vm_1 + 2(1+\upsilon)m_2{}^2]. \quad (32)$$

Substitute Eq.21 and Eq.22 into Eq.24 yields

$$m_1 = \frac{\sigma_y}{\sigma_x} = \frac{-\frac{EAZ}{(1-\upsilon^2)a^2}\left(\upsilon\frac{\partial^2 h}{a^2\,\partial R^2} + \frac{1}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)}{-\frac{EAZ}{(1-\upsilon^2)a^2}\left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)}. \quad (33)$$

$$m_1 = \frac{\left(\upsilon\frac{\partial^2 h}{a^2\,\partial R^2} + \frac{1}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)}{\left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)}. \quad (34)$$

Substitute Eq.21 and Eq.23 into Eq.25 yields

$$m_2 = \frac{\tau_{xy}}{\sigma_x} = \frac{-\frac{EAZ(1-\upsilon)}{2a^2(1-\upsilon^2)Ƨ}\left(\frac{\partial^2 h}{\partial R\,\partial Q}\right)}{-\frac{EAZ}{(1-\upsilon^2)a^2}\left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)}. \quad (35)$$

$$m_2 = \frac{(1-\upsilon)}{2Ƨ}\,\frac{\left(\frac{\partial^2 h}{\partial R\,\partial Q}\right)}{\left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)}. \quad (36)$$

Eq.34 and Eq.36 becomes Eq.37 and Eq.38 respectively.

$$m_1 = \frac{n_2}{n_1}. \quad (37)$$

$$m_2 = \frac{(1-\upsilon)}{2\text{ƨ}}\frac{n_3}{n_1}. \qquad (38)$$

Where,

$$n_1 = \frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{\text{ƨ}^2}\frac{\partial^2 h}{\partial Q^2} = \frac{\partial^2 h_x}{\partial R^2} * h_y + h_x * \frac{\partial^2 h_y}{\partial Q^2}. \qquad (39)$$

$$n_2 = \frac{\upsilon}{a^2}\frac{\partial^2 h}{\partial R^2} + \frac{1}{\text{ƨ}^2}\frac{\partial^2 h}{\partial Q^2} = \frac{\upsilon}{a^2}\left(\frac{\partial^2 h_x}{\partial R^2} * h_y\right) + \frac{1}{\text{ƨ}^2}\left(h_x * \frac{\partial^2 h_y}{\partial Q^2}\right). \qquad (40)$$

$$n_3 = \frac{\partial^2 h}{\partial R\,\partial Q} = \frac{\partial h_x}{\partial R} * \frac{\partial h_y}{\partial Q}. \qquad (41)$$

Eq.39, Eq.40 and Eq.41 are the general n-value equations as presented in Table-2.

Substituting Eq.39, Eq.40 and Eq.41 into Eq.31 yields

$$F = \sqrt{\left[1 + \frac{{n_2}^2}{{n_1}^2} - 2\upsilon\frac{n_2}{n_1} + \frac{(1+\upsilon)(1-\upsilon)^2}{2\text{ƨ}^2}\frac{{n_3}^2}{{n_1}^2}\right]}. \qquad (42)$$

Simplifying we have

$$F = \sqrt{\left[1 + \frac{{n_2}^2}{{n_1}^2} - 2\upsilon\frac{n_2}{n_1} + \frac{(1+\upsilon)}{2}\left(\frac{1-\upsilon}{\text{ƨ}}\right)^2\frac{{n_3}^2}{{n_1}^2}\right]}. \qquad (43)$$

Again, evaluate further yields

$$F = \sqrt{\left[1 + \frac{{n_2}^2}{{n_1}^2} - 2\upsilon\frac{n_2}{n_1} + \frac{(1-\upsilon-\upsilon^2+\upsilon^3)}{2\text{ƨ}^2}\frac{{n_3}^2}{{n_1}^2}\right]}. \qquad (44)$$

Substituting Eq.44 into Eq.30 becomes

$$\sigma_x \le \frac{f_y}{\sqrt{\left[1 + \frac{{n_2}^2}{{n_1}^2} - 2\upsilon\frac{n_2}{n_1} + \frac{(1-\upsilon-\upsilon^2+\upsilon^3)}{2\text{ƨ}^2}\frac{{n_3}^2}{{n_1}^2}\right]}}. \qquad (45a)$$

$$\sigma_x \le \frac{f_y}{\sqrt{\left[\frac{{n_1}^2 + {n_2}^2 - 2\upsilon n_1 n_2 + \frac{(1-\upsilon-\upsilon^2+\upsilon^3)}{2\text{ƨ}^2}{n_3}^2}{{n_1}^2}\right]}}. \qquad (45b)$$

Note: if $\sigma_x < f_y$ (Elastic limit); $\sigma_x = f_y$ (Purely Plastic ie yield has ocurred);
$\sigma_x > f_y$ (Not attainable)

Eq.45 is the general yield criterion equation.

The general formulated Allowable stress Equation and Stress Factor Equation (that is Eq.45 and Eq.44 respectively) are presented in Table-2.

C. Evaluation of 'n-Values' and Formulation of 'n-Values' Equations

The various n-values (that is, n_1, n_2, and n_3) for the different plate types will be evaluated using the polynomial displacement shape profiles in Table-1.

Table-1 The polynomial displacement shape profiles (Ibearugbulem *et al.*, 2014)

Plate Type	Shape Profile, h
SSSS	$(R-2R^3+R^4)(Q-2Q^3+Q^4)$
CCCC	$(R^2-2R^3+R^4)(Q^2-2Q^3+Q^4)$
CSSS	$(R-2R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
CSCS	$(R-2R^3+R^4)(Q^2-2Q^3+Q^4)$
CCSS	$(1.5R^2-2.5R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
CCCS	$(1.5R^2-2.5R^3+R^4)(Q^2-2Q^3+Q^4)$
SSFS	$(R\text{-}2R^3+R^4)\ (\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
SCFS	$(1.5R^2\text{-}2.5R^3+R^4)\ (\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
CSFS	$(R\text{-}2R^3+R^4)\ (2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
CCFS	$(1.5R^2\text{-}2R^3+R^4)\ (2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
SCFC	$(R^2\text{-}2R^3+R^4)\ (\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
CCFC	$(R^2\text{-}2R^3+R^4)\ (2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$

S- Simply supported edge, C - Clamped edge, F - Free edge

Where,

SSSS - a plate simply supported on all the four edges

CCCC- a plate clamped/fixed on all the four edges, and so on.

R = X/a, 0≤R≤1; Q= Y/b, 0≤R≤1

a - plate dimension (length) along X-axis, b - is plate dimension (Width) along Y-axis

The n-values for the various plate types will be evaluated as follows.

D. Evaluation of n-Values for SSSS Plate

From Table 1,

$$h = (R-2R^3+R^4)(Q-2Q^3+Q^4) = h_x * h_y. \tag{46}$$

$$\frac{\partial^2 h}{\partial R^2} = \frac{\partial^2 h_x}{\partial R^2} * h_y = (-12R^2+12R^2)(Q-2Q^3+Q^4). \tag{47a}$$

$$\frac{\partial^2 h}{\partial Q^2} = h_x * \frac{\partial^2 h_y}{\partial Q^2} = (R-2R^3+R^4)(-12Q+12Q^2). \tag{47b}$$

$$\frac{\partial^2 h}{\partial R\,\partial Q} = \frac{\partial h_x}{\partial R} * \frac{\partial h_y}{\partial Q} = (-6R^2+4R^3)(-6Q^2+4Q^3). \tag{47c}$$

Substitute Equations (47) into Equations (39)- (41) yields

$$n_1 = \Big[(-12R^2+12R^2)(Q-2Q^3+Q^4) + \frac{\upsilon}{\text{Ƨ}^2}(R-2R^3+R^4)(-12Q+12Q^2)\Big]. \tag{48}$$

$$n_2 = \Big[\upsilon(-12R^2+12R^2)(Q-2Q^3+Q^4) + \frac{1}{\text{Ƨ}^2}(R-2R^3+R^4)(-12Q+12Q^2)\Big]. \tag{49}$$

$$n_3 = (1-6R^2+4R^3)(1-6Q^2+4Q^3) \tag{50}$$

At the point of maximum deflection, R = Q = 0.5. Substitute these values of R and Q in Eq.48 to Eq.50, we have

$$n_1 = \left(-0.9375 - \frac{0.9375\upsilon}{\text{Ƨ}^2}\right). \tag{51}$$

$$n_2 = \left(-0.9375\upsilon - \frac{0.9375}{\text{Ƨ}^2}\right). \tag{52}$$

$$n_3 = 0. \tag{53}$$

Substituting Eq.51 to Eq.53 into Eq.44 yields the stress factor as Eq.54

$$F = \left[1 + \frac{\left(-0.9375\upsilon - \frac{0.9375}{ƨ^2}\right)^2}{\left(-0.9375 - \frac{0.9375\upsilon}{ƨ^2}\right)^2} - 2\upsilon \frac{\left(-0.9375\upsilon - \frac{0.9375}{ƨ^2}\right)}{\left(-0.9375 - \frac{0.9375\upsilon}{ƨ^2}\right)}\right]^{\frac{1}{2}}. \qquad (54)$$

Similarly, the rest of the 11 plate types contain in Table 1, were evaluated.
The n-values equations (that is, Eq.51 to Eq.53) for the SSSS plate and other plate types under consideration are presented in Table-3. While stress factor equations for the twelve plates are presented in Table-4.

E. Numerical Application
Consider a structural steel square plate with the following properties. $\upsilon = 0.3$, a =1m, f_y =250MPa.
The numerical results obtain from yield criterion equations in Table-3-Table-4 are presented in Table-5.

RESULTS AND DISCUSSION

The popular widely used yield criteria are those of Trisca and von Mises. In this work, a new yield criterion approach is presented. The general yield criterion or allowable stress equation is given by Eq.45 and presented in row one of Table-2. Also, the general n-values equations are given in Eq.39 to Eq.41 and the stress factor equation given by any of the Eq.42, Eq.43 or Eq.44 are presented in row three and row two of Table-2 respectively. The general equation here applies to any boundary condition and any thin structural plate material. The specific n-values equations and stress factor equations for the various plate types under consideration are presented in Table-3 and Table-4 respectively. This is the first time these equations are given for the analysis of allowable stress within and beyond the elastic limit.

Table-2 General Formulated Yield Criterion Equations

SN	DESCRIPTION	EQUATIONS
1	Allowable Stress	$\sigma_x \leq \frac{f_y}{F}$
2	Stress Factor of Safety	$F = \frac{1}{n_1}\sqrt{\left[{n_1}^2 + {n_2}^2 - 2vn_1n_2 + \frac{(1+\upsilon)(1-\upsilon)^2}{2ƨ^2}{n_3}^2\right]}$
3	n-values	$n_1 = \left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)$ $n_2 = \left(\upsilon\frac{\partial^2 h}{\partial R^2} + \frac{1}{ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)$ $n_3 = \left(\frac{\partial^2 h}{\partial R\,\partial Q}\right)$

Table-3 n-value Equations for Failure Analysis at the point of Maximum Deflection

Plate Type	$n_1 = \left(\frac{\partial^2 h}{\partial R^2} + \frac{\upsilon}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)$	$n_2 = \left(\upsilon\frac{\partial^2 h}{\partial R^2} + \frac{1}{Ƨ^2}\frac{\partial^2 h}{\partial Q^2}\right)$	$n_3 = \left(\frac{\partial^2 h}{\partial R\,\partial Q}\right)$
SSSS	$\left(-0.9375 - \frac{0.9375\upsilon}{Ƨ^2}\right)$	$\left(-0.9375\upsilon - \frac{0.9375}{Ƨ^2}\right)$	0
CCCC	$\left(-0.0625 - \frac{0.0625\upsilon}{Ƨ^2}\right)$	$\left(-0.0625\upsilon - \frac{0.0625}{Ƨ^2}\right)$	0
CSSS	$\left(-0.375 - \frac{0.46875\upsilon}{Ƨ^2}\right)$	$\left(-0.375\upsilon - \frac{0.46875}{Ƨ^2}\right)$	0
CSCS	$\left(-0.1875 - \frac{0.3125\upsilon}{Ƨ^2}\right)$	$\left(-0.1875\upsilon - \frac{0.3125}{Ƨ^2}\right)$	0
CCSS	$\left(-0.1875 - \frac{0.1875\upsilon}{Ƨ^2}\right)$	$\left(-0.1875\upsilon - \frac{0.1875}{Ƨ^2}\right)$	0.015625
CCCS	$\left(-0.09375 - \frac{0.125\upsilon}{Ƨ^2}\right)$	$\left(-0.09375\upsilon - \frac{0.125}{Ƨ^2}\right)$	0
SSFS	-4	-4υ	0
SCFS	-2	-2υ	0.083333333
CSFS	-1.2	-1.2υ	0
CCFS	-0.6	-0.6υ	0.025
SCFC	-1.333333333	-1.333333333υ	0
CCFC	-0.4	-0.4υ	0

Table-4 Stress Factor, F, Equations

Plate Type	$\sigma_x \leq \frac{f_y}{F}$
	$F = \sqrt{\left[1 + \frac{{n_2}^2}{{n_1}^2} - 2\upsilon\frac{n_2}{n_1} + \frac{(1-\upsilon-\upsilon^2+\upsilon^3)}{2ƨ^2}\frac{{n_3}^2}{{n_1}^2}\right]}$
SSSS	$\left[1 + \frac{(-0.9375ƨ^2\upsilon - 0.9375)^2}{(-0.9375ƨ^2 - 0.9375\upsilon)^2} - 2\upsilon\frac{(-0.9375ƨ^2\upsilon - 0.9375)}{-0.9375ƨ^2 - 0.9375\upsilon}\right]^{\frac{1}{2}}$
CCCC	$\left[1 + \frac{(-0.0625ƨ^2\upsilon - 0.0625)^2}{(-0.0625ƨ^2 - 0.0625\upsilon)^2} - 2\upsilon\frac{(-0.0625ƨ^2\upsilon - 0.0625)}{(-0.0625ƨ^2 - 0.0625\upsilon)}\right]^{\frac{1}{2}}$
CSSS	$\left[1 + \frac{(-0.375ƨ^2\upsilon - 0.46875)^2}{(-0.375ƨ^2 - 0.46875\upsilon)^2} - 2\upsilon\frac{(-0.375ƨ^2\upsilon - 0.46875)}{(-0.375ƨ^2 - 0.46875\upsilon)}\right]^{\frac{1}{2}}$
CSCS	$\left[1 + \frac{(-0.1875ƨ^2\upsilon - 0.3125)^2}{(-0.1875ƨ^2 - 0.3125\upsilon)^2} - 2\upsilon\frac{(-0.1875ƨ^2\upsilon - 0.3125)}{(-0.1875ƨ^2 - 0.3125\upsilon)}\right]^{\frac{1}{2}}$
CCSS	$\left[1 + \frac{(-0.1875ƨ^2\upsilon - 0.1875)^2}{(-0.1875ƨ^2 - 0.1875\upsilon)^2} - 2\upsilon\frac{(-0.1875ƨ^2\upsilon - 0.1875)}{(-0.1875ƨ^2 - 0.1875\upsilon)} + \frac{0.0001220703125ƨ^2(1-\upsilon-\upsilon^2+\upsilon^3)}{(-0.1875ƨ^2 - 0.1875\upsilon)^2}\right]^{\frac{1}{2}}$
CCCS	$\left[1 + \frac{(-0.09375ƨ^2\upsilon - 0.125)^2}{(-0.09375ƨ^2 - 0.125\upsilon)^2} - 2\upsilon\frac{(-0.09375ƨ^2\upsilon - 0.125)}{(-0.09375ƨ^2 - 0.125\upsilon)}\right]^{\frac{1}{2}}$
SSFS	$[1 + \upsilon^2 - 2\upsilon^2]^{\frac{1}{2}}$
SCFS	$\left[1 + \upsilon^2 - 2\upsilon^2 + \frac{0.0008680555486(1-\upsilon-\upsilon^2+\upsilon^3)}{ƨ^2}\right]^{\frac{1}{2}}$
CSFS	$[1 + \upsilon^2 - 2\upsilon^2]^{\frac{1}{2}}$
CCFS	$\left[1 + \upsilon^2 - 2\upsilon^2 + \frac{0.0008680555556(1-\upsilon-\upsilon^2+\upsilon^3)}{ƨ^2}\right]^{\frac{1}{2}}$
SCFC	$[1 + \upsilon^2 - 2\upsilon^2]^{\frac{1}{2}}$
CCFC	$[1 + \upsilon^2 - 2\upsilon^2]^{\frac{1}{2}}$

Table-5 shows the numerical results for n-values as presented in columns 2, 3, and 4 for n_1, n_2, and n_3 respectively. While the results for the stress factor of safety for the various plate types are presented in column 5 of Table-5. And the allowable stress values using the yield stress of 250MPa for mild steel are presented in column 6. Generally, when the yield consideration is within the elastic limit, the actual yield stress f_y for mild steel is taken as 250MPa. Beyond this stress, initial yield is prominent within the elastic range.

The numerical results from Table-5, indicate that the stress factor for plates without any free edge is above unity while for plates with one free edge are below unity but approximately equal to one. The allowable stress values obtained using f_y =250MPa (considering yield within elastic range) imply that beyond these values the plate will yield. These results are in line with those of existing yield criterion equations expressed in pieces of literature by Lee and Ross. So, from the predicted values of allowable stress from the new equation in this work for the various plate types considered as seen in column 6 of the Table 5, it is shown that, when the stress factor of safety is less than unity, the stress is above 250MPa. That is, the likelihood of failure or yield is prominent.

Based of the above observation, for general thin rectangular plates design, taking the average of the stress factor values, a conservative stress factor can be taken as 1.10. Therefore, for design purposes, the general stress factor for all thin plates based on the numerical results obtain here may be taken as 1.10. These yield values are in tandem with ranges of the initial or minimum yield stress of Grade 43 steel (S275) given by BS EN 10025:1993. But when the failure is within the inelastic range, then the rupture or ultimate stress is considered and according to Gere (2004), the ultimate stress for mild A36 mild steel is 400MPa. The allowable yield stress becomes limit state stress if the ultimate yield stress (fy = 400MPa) of a material is used instead of the initial yield stress (fy = 250MPa). This work has set the limit of stress that should be allowed if failure or yield of rectangular plate structures must be avoided. This will reduce the incidence of plated structure failure, and save resources and economic damages. Beyond this, the formulated equations provide a quicker means of analysis of yield in rectangular plates, and will be most beneficial to plate analyst and designers, and the aerospace and shipbuilding industries.

Table-5 Allowable Stress at from Failure Criterion Analysis

Plate type	$\sigma_{allow} \leq \frac{f_y}{F}$; $\varepsilon = \frac{b}{a} = 1$; $f_y = 250MPa$				
	n_1	n_2	n_3	F	σ_{allow}(MPa)
SSSS	-1.21875	-1.21875	0	1.183216	211.2886
CCCC	-0.08125	-0.08125	0	1.183216	211.2886
CSSS	-0.51563	-0.58125	0	1.262688	197.9904
CSCS	-0.28125	-0.36875	0	1.390088	179.8447
CCSS	-0.24375	-0.24375	0.015625	1.183769	211.1899
CCCS	-0.13125	-0.15313	0	1.288841	193.9727
SSFS	-4	-1.2	0	0.953939	262.0712
SCFS	-2	-0.6	0.083333	0.954229	261.9916
CSFS	-1.2	-0.36	0	0.953939	262.0712
CCFS	-0.6	-0.18	0.025	0.954229	261.9916
SCFC	-1.33333	-0.4	0	0.953939	262.0712
CCFC	-0.4	-0.12	0	0.953939	262.0712

CONTRIBUTION TO KNOWLEDGE

This work has formulated a new general allowable stress equation, n-value equations and stress factor equation for the yield or failure criterion analysis of thin rectangular plates. It has also, formulated specific allowable stress equations, n-value equations and stress factor equations for twelve (12) thin rectangular plate types. These equations are applicable to any edge condition of plate. These new equations will no doubt aid easy analysis and design of thin rectangular plates, save time, energy and economic damage. This work will open up a new dimension to research in this field.

CONCLUSION

This study has formulated a new general allowable stress equation for yield analysis of thin rectangular plates that will simplify the solution to yield problems. It has shown that the stress factor of safety of plated structures can be taken as 1.10 based on the average values predicted from the new equation for various plate types considered. Based on this work, the limit of the safety of plated structures is set, and beyond which the probability of failure or yield becomes imminent. Through this work, they will be a reduction in occurrences of thin plate structures failure, saving of resources, and economic damages. This is a new dimension to the field of yield criterion that will be relevant to analysts and designers of plated structures.

CONFLICT OF INTEREST

There is no conflict of interest for this research work.

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