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Experimental Investigation and Simulation of Lead Extrusion Process for Complex Shape

1*Enahoro Micheal Oamhen and 2Irogue Wilfred Aiguobasimwin

1 Department of Mechanical Engineering, University of Benin, Benin City, Edo State, Nigerian
micheal.enahoro@uniben.edu

2Department of Materials and Metallurgical Engineering, University of Benin, Benin City, Edo State, Nigerian
wilfred.irogue@uniben.edu

**Corresponding Author: Enahoro Micheal Oamhen;* micheal.enahoro@uniben.edu

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Abstract: *This is an analytical and experimental research study of cold extrusion of lead billets through complex E and X-shaped dies based on the upper bound theorem method, relying on predictive analysis. The purpose of the research is to assess the potential of the upper bound analysis of the determination of critical extrusion parameters, i.e. forming load, pressure, effective strain, and strain rate. A theory of energy dissipation based on theoretically determined kinematically admissible velocity fields was worked out, including the contribution of the internal deformation work, the losses through friction and interface shear. To examine how interfacial friction affects the performance of extrusion, four types of lubrication conditions were taken: no lubricant, red oil, brake oil, and mustard oil. A hydraulic press and precision-machined steel dies were used in the conduct of experimental trials, and data on the forming loads and displacements were measured by hand. The finite element modelling was also used to carry out the extrusion simulation and verification of theoretical assumptions. The results were such that prognostic predictions made using the analysis, the experiment, and the FEM were well matched, and the extrusion load difference was 3.7-6.7 % in reversion. Brake oil, being of the highest viscosity, reduced forming loads consistently in support of the model as regards the lubricant effect on forming loads. The findings substantiate the idea that hybrid analytical-numerical tools are of enormous value in terms of developing concepts of imagination of deformation processes and manufacturing strategy on the industrial level.*

Keywords: Lead Extrusion, Simulation, Complex Shapes, Finite Element Model, Investigation

INTRODUCTION

The prediction of the loads that will be formed and the material flow during the metal extrusion is very important as goes as far as the design and optimisation of the metal forming processes are concerned (Qamar *et al.,* 2019). As more complex extrusion processes are being developed (to produce parts whose shapes are more complex), a way to predict their deformation properties and what the process has to achieve those properties is required. To such tools, one might count the upper bound theorem in plasticity-based modelling because this tool has been used to get quite corroborating estimates of forces necessitated and energy usage in forming processes (Zhang *et al.,* 2023). The upper bound method, based on limit analysis, gives the largest rate of internal work that can be maintained by a prescribed set of kinematically permissible velocity fields and, as such, provides a theoretical upper bound of the forming load necessary to plastically deform a workpiece.

Although being a most popular topic of metal forming research, the practical use of the upper bound theorem in metal forming processes, including through complex extrusion shapes, is frequently limited by the ability to achieve the appropriate velocity, strain rate fields that meet boundary and incompressibility conditions (Zhang *et al.,* 2023). Additionally, in most of the traditional upper bound models, conditions are assumed to be ideal, and this may create differences when the model results are compared with actual operations, e.g. material properties are considered as homogeneous, plasticity as perfect and simplification of interaction between tool and workpiece. In extrusion processes with asymmetric or non-uniform shaped extrudates like the 'E' and 'X' shaped extrudates, these assumptions are ever more important to test with experimental as well the numerical verification (Oyinbo *et al.,* 2015).

Other writers have explored the hybridisation of analytical solutions in its upper-bound manifestation with numerical techniques like finite element analysis (Yahaya *et al.,* 2023) or other kinds of experimental processes, to fill up this vacuum. For instance, Farzad and Ebrahimi (2017) as demonstrated in this paper, the upper bound approach can be applied to plane-strain extrusion and provide the accuracy of the predicted load by solutions that are 10 per cent of those of the FEM simulation solutions. Likewise, utilised the method to examine the effect of friction and die geometry on pressure in extrusion, which added more credence to it using the lab one. (Li *et al.,* 2022). It is postulated in all these studies that the upper bound technique, though crude in nature, can provide results which are accurate enough, provided it is calibrated and contextualised. An upper bound theorem is used in this paper to formulate the expressions of extrusion pressure, plastic strain, strain rate and as well as power dissipation of cold extrusion of lead billets into complex shapes. The modelling framework utilised covers the loss of friction, the die geometry and assumptions of material flow in the process of extrusion of E and X Shapes (Shahbaz *et al.,* 2016). Metal forming is at the heart of the current manufacturing processes as it allows the efficient realisation of complex shapes with higher mechanical properties. These are forms like extrusion, forging, and rolling, and they depend strongly on the plastic deformation of metals under induced loads, usually at higher temperatures. But the development of more complex shapes of components and the need to develop higher precision in forming operations require complex computational tools that would simulate and optimise forming processes. This experiment, therefore, adds value to the ongoing dialogue about analytical-experimental model integration in metal forming and provides a hands-on study of employing upper-bound principles to do quick process estimation and design iteration in the industrial context.

MATERIALS AND METHODS

In this experiment, plain carbon steel material was machined and shaped to various parts such as container, dies and punch. The dies where the spacklings were created and the punch were hardened to increase the strength of the material. Lead metal was carefully chosen, to form the required metal shape due to its favorable properties to achieve better result for this research. Scraps of lead were melted to recast a cylindrical rod shape and machined to requisite dimension for the purpose of the testing. With the completed tools and required available materials ready, the experiment was carried out using the extrusion machine. Results were collected for the appropriate analysis. The chosen material for the tools was plain carbon steel. The container, the punch and the dies were all machined to the required specifications (Fig. 1a, 1b, 1c & 1d) using the lathe machine. 'E' & 'X' shapes sparkling on the dies was carried out with the CNC machine. The designed container is a rectangular block, made of plain carbon steel. It serves as the housing for all other components which includes the 'E' & 'X' shape dies and the punch that were used for the experiment. It was designed in such a way that, it could be opened when off-loading the completed formed metals and closed using bolts to effectively conduct test. Figs. 2 and 3 show the pictorial view of the uncoupled and coupled tools respectively.

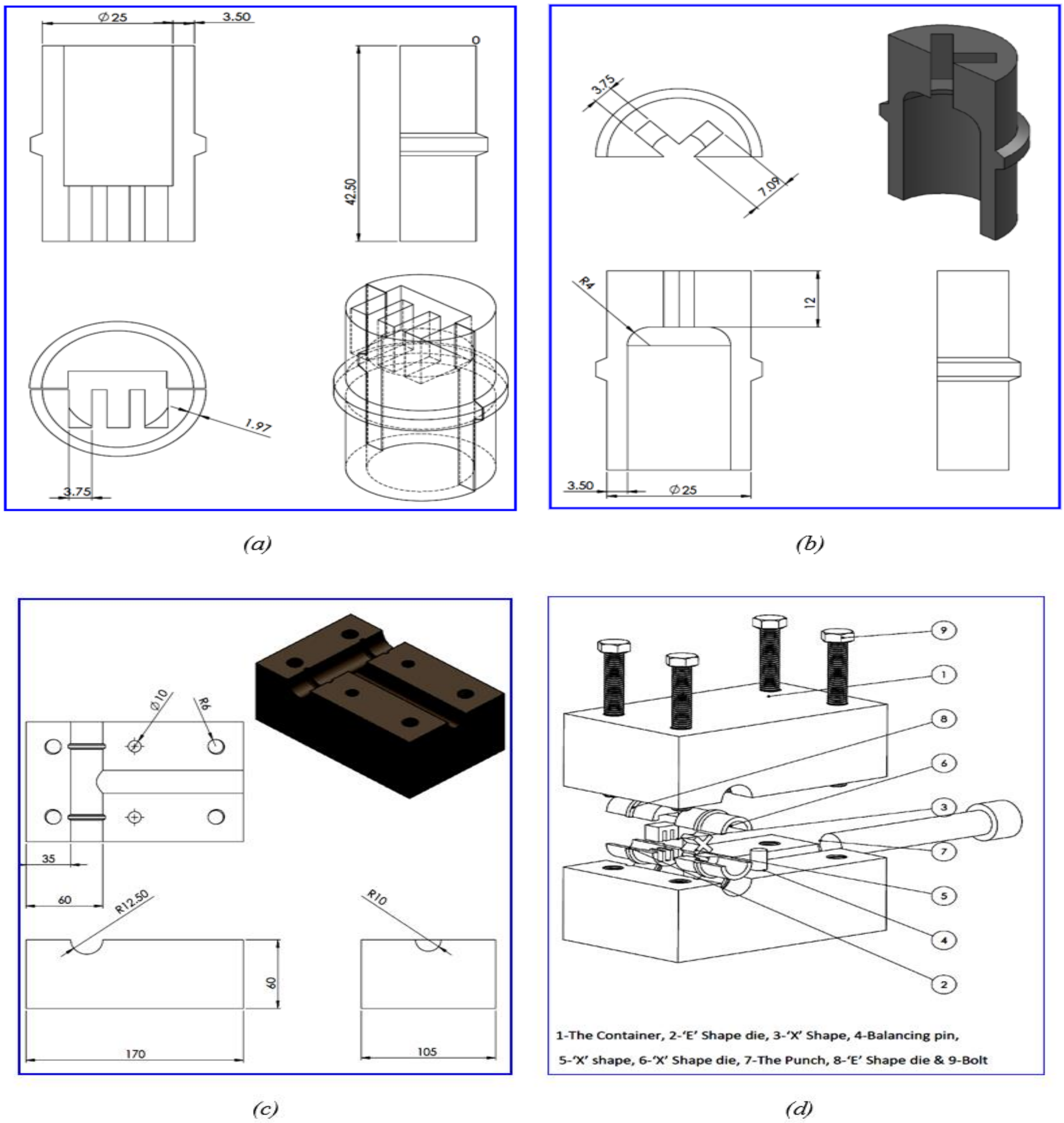


Fig. 1 *the Drawings (a) 'E' Shape die, (b) 'X' Shape die, (c) Half part of the container (d) Coupled tool*

Fig. 2 *Pictorial view of the uncoupled experimental tools*

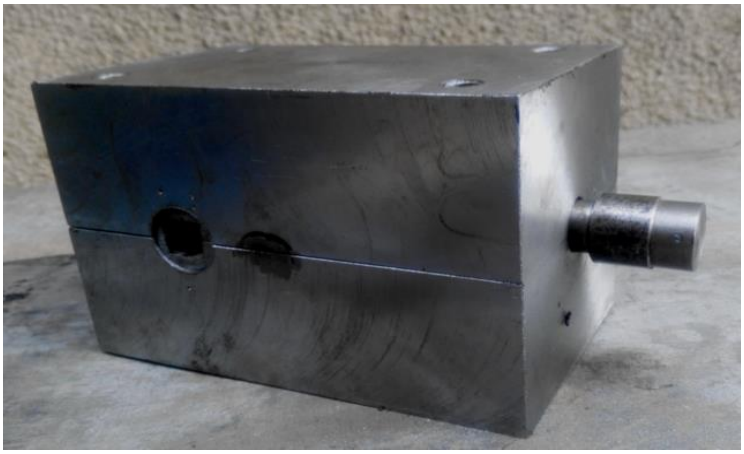


Fig. 3 Pictorial view of the coupled tools

Pieces of lead materials were sourced, remolded and casted to the required sizes. The casted lead metal was machined to needed dimensions of cylindrical shapes of 20mm diameter by 55 mm height. With the container, dies, punch and the billets ready, the components are coupled together as shown in Fig. 3. Using the extrusion equipment (Fig. 4), load is applied to the punch, which in turn pushes the billet that tends to flow while deforming through the available spaces in the dies. As the load travels in the vertical direction, the billet deforms from the same direction to flow through the shapes on the dies in the lateral and in opposite directions to form the shapes sparkled on the dies. The distant travelled by the applied loads at each point and time were taken and recorded accordingly (Fig. 4). The experiment was repeated at without/with red, brake and mustard oil.

Fig. 4 Taking Experimental Readings

It was observed that, the effect of friction between the container and the work piece as well as that with the dies was reduced on lubrication. It was further detected that, the higher the fluid viscosity the more effective the lubricant on the frictional effect. To form the 'E' & 'X' Shapes, the lead billet is fed into the container, and with load applied to it through the punch in the vertical direction, it to flows laterally and in opposite directions to form the 'E' and 'X' shapes on the respective dies placed appropriately.

(a) (b) (c)

Fig. 5 Formed Shapes (a)Formed 'E' Shape, (b) Formed 'X' Shape & (c) Finished formed 'E' & 'X' Shapes

RESULTS AND DISCUSSION

The results of the corresponding displacements, forces and times were recorded and tabulated appropriately (Table-1).

Table-1 Experimental Results(a)No lubricant, (b) Red oil as lubricant, (c) Brake oil as lubricant & (d) Mustard oil as lubricant

Final Data for No Lubricant					
X (mm)	F x10^6 N	T (Sec)	V(m/s)	ε-(ΔH/H)	σ x10^3 N/m^2
0.00	0.00	0.00	0.00	0.00	0.00
0.50	14.00	5.47	9.14	0.01	44.55
1.00	36.00	21.55	3.11	0.02	114.55
1.50	54.50	43.74	2.25	0.03	173.41
2.00	67.50	68.21	2.04	0.04	214.77
2.50	78.50	89.56	2.34	0.05	249.77
3.00	86.75	123.61	1.47	0.05	276.02
3.50	88.50	153.51	1.67	0.06	281.59
4.00	87.75	175.68	2.26	0.07	279.20
4.50	87.75	196.35	2.42	0.08	279.20
5.00	87.75	237.43	1.22	0.09	279.20
5.50	87.75	257.97	2.43	0.10	279.20
Average Velocity			2.76	Ratio (E:X)	
Average Shapes Length		E	29.38	0.54	Original Height (mm)
		X	25.50	0.46	55.00

(a)

Final Data for Red Oil Lubricant					
X (mm)	F x10^6 N	T (Sec)	V(m/s)	ε-(ΔH/H)	σ x10^3 N/m^2
0.00	0.00	0.00	0.00	0.00	0.00
0.50	23.50	14.55	3.55	0.01	74.77
1.00	35.00	31.96	2.90	0.02	111.36
1.50	43.50	50.81	2.67	0.03	138.41
2.00	54.50	71.07	2.48	0.04	173.41
2.50	61.00	92.68	2.34	0.05	194.09
3.00	63.50	111.67	2.63	0.05	202.05
3.50	66.00	135.12	2.13	0.06	210.00
4.00	65.50	153.57	2.98	0.07	208.41
4.50	65.00	171.87	2.79	0.08	206.82
5.00	65.00	191.04	2.64	0.09	206.82
5.50	65.00	219.34	1.92	0.10	206.82
Average Velocity			2.64	Ratio (E:X)	
Average Shapes Length		E	21.38	0.55	Original Height (mm)
		X	17.50	0.45	55.00

(b)

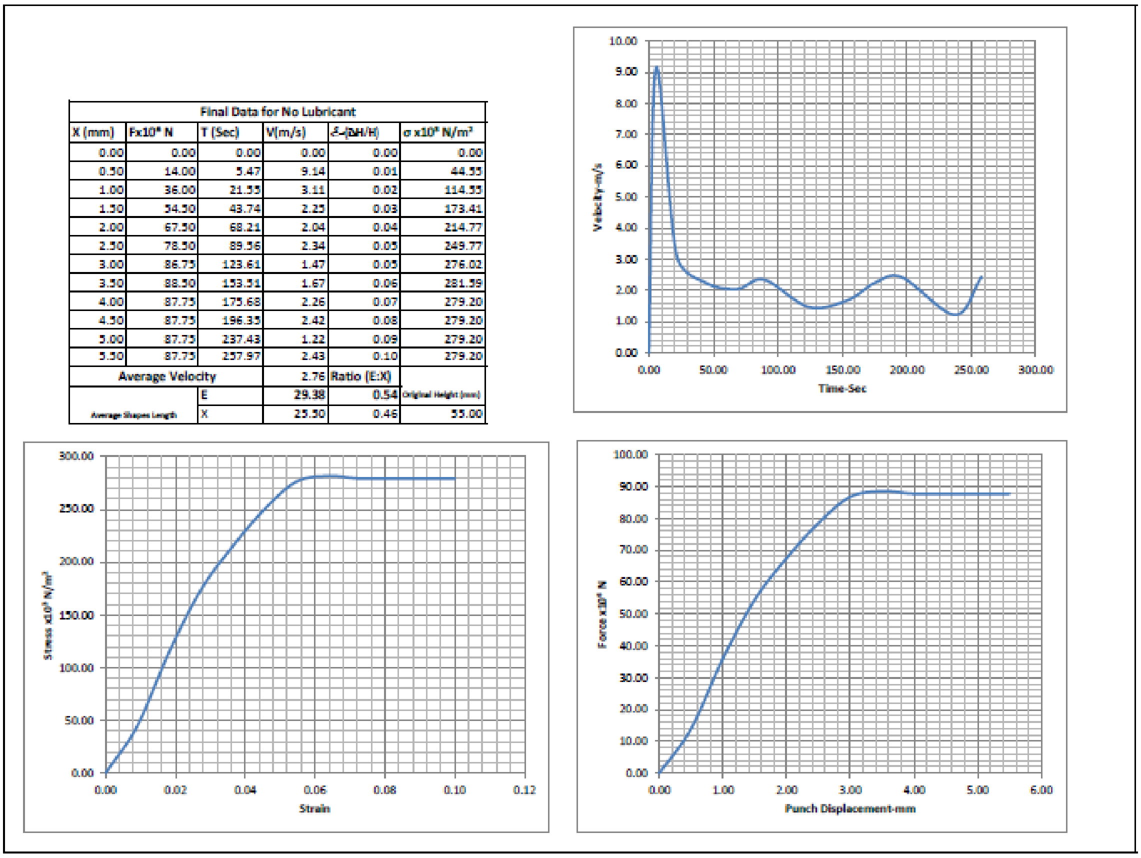
Final Data for Brake Oil Lubricant					
X (mm)	F x10^6 N	T (Sec)	V(m/s)	ε-(ΔH/H)	σ x10^3 N/m^2
0.00	0.00	0.00	0.00	0.00	0.00
0.50	20.50	4.12	12.23	0.01	65.23
1.00	36.50	18.99	3.52	0.02	116.14
1.50	46.50	38.33	2.59	0.03	147.95
2.00	54.50	57.55	2.60	0.04	173.41
2.50	60.00	78.24	2.42	0.05	190.91
3.00	63.00	98.98	2.41	0.05	200.45
3.50	65.00	124.32	1.97	0.06	206.82
4.00	64.00	144.23	2.54	0.07	203.64
4.50	62.50	161.84	2.90	0.08	198.86
5.00	62.50	179.37	2.87	0.09	198.86
5.50	62.50	205.46	1.99	0.10	198.86
Average Velocity			3.46	Ratio (E:X)	
Average Shapes Length		E	21.00	0.55	Original Height (mm)
		X	17.50	0.45	55.00

(c)

Final Data for Mustard Oil Lubricant					
X (mm)	F x10^6 N	T (Sec)	V(m/s)	ε-(ΔH/H)	σ x10^3 N/m^2
0.00	0.00	0.00	0.00	0.00	0.00
0.50	27.00	7.36	8.93	0.01	85.91
1.00	40.00	25.33	2.78	0.02	127.27
1.50	48.50	41.66	3.15	0.03	154.32
2.00	55.00	65.63	2.16	0.04	175.00
2.50	59.50	87.11	2.33	0.05	189.32
3.00	63.50	108.61	2.33	0.05	202.05
3.50	66.00	130.11	2.33	0.06	210.00
4.00	69.50	151.23	2.38	0.07	221.14
4.50	67.00	168.25	2.98	0.08	213.18
5.00	66.00	189.38	2.37	0.09	210.00
5.50	66.00	203.10	4.22	0.10	210.00
Average Velocity			3.27	Ratio (E:X)	
Average Shapes Length		E	21.38	0.55	Original Height (mm)
		X	17.50	0.45	55.00

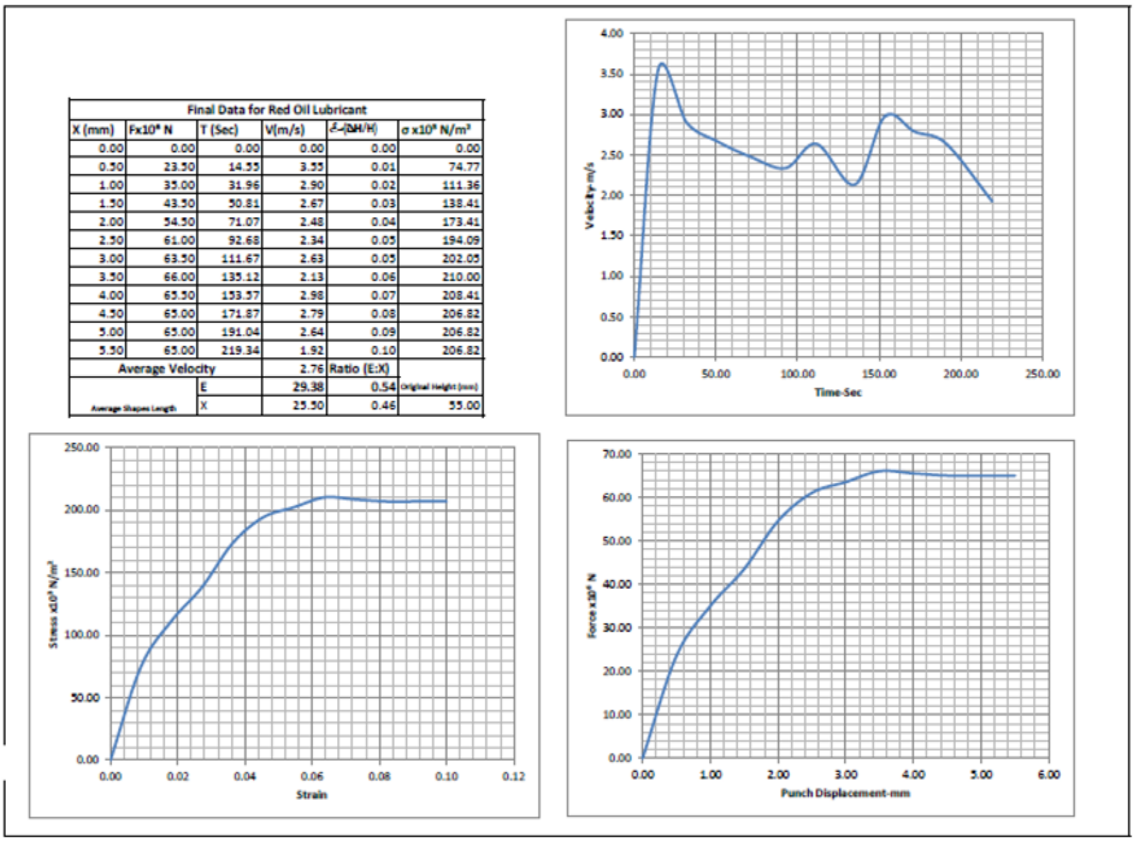
(d)

The graphs shown in Figs. 6-9 below explains the behaviour of forces, stresses and velocities at no lubricant, using red, brake and mustard oil respectively. In each case, the performance of forces on displacements and stresses against strains were the same. However, the activities were slightly different from one operational condition to the other. The analysis showed that, the maximum load and corresponding extrusion pressure required to ensure a complete cycle varies from 88.5, 66.00, 65.00 and 69.5MN and 281.59, 210.00, 206.82 and 221.14KN/ m² during the experiment at no lubrication, red, brake and mustard oil respectively. This agrees with the initial submission during the experimental observation on the effect of friction, that, the higher the fluid viscosity, the lesser load required to exert pressure on the cross-sectional surface of the lead billets. Unlike the behavior of forces and stresses on displacements and strains respectively, the graphs in Figs. 6-9 show that, the lubricant did not determine the pattern of the velocity for this experiment. Whereas, the average velocity recorded when lubricant was not applied was 2.76m/s, but for red, brake and mustard oils were 2.64, 3.46 and 3.27m/s accordingly. It therefore means, further investigation will be required to ascertain the effect of lubricant on velocity.



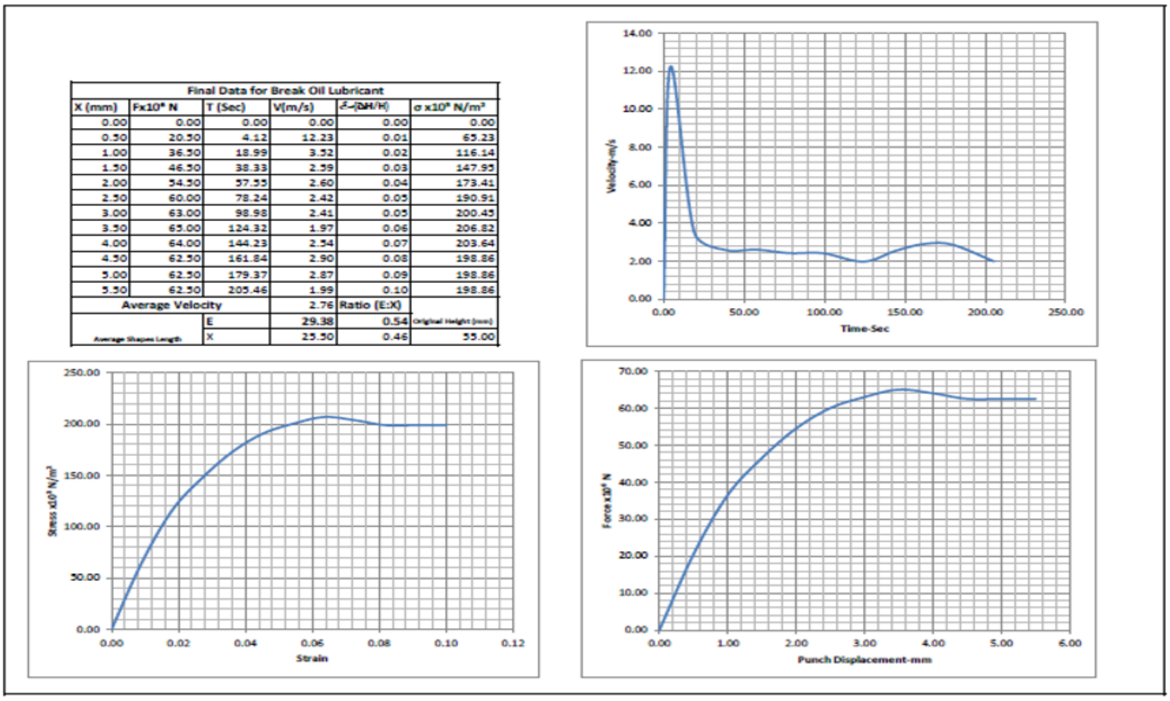
Final Data for No Lubricant					
X (mm)	Fx10^{6} N	T (Sec)	V(m/s)	Ꜫ-(ΔH/H)	σ x10^{3} N/m²
0.00	0.00	0.00	0.00	0.00	0.00
0.50	14.00	5.47	9.14	0.01	44.55
1.00	36.00	21.55	3.11	0.02	114.55
1.50	54.50	43.74	2.25	0.03	173.41
2.00	67.50	68.21	2.04	0.04	214.77
2.50	78.50	89.56	2.34	0.05	249.77
3.00	86.75	123.61	1.47	0.05	276.02
3.50	88.50	153.51	1.67	0.06	281.59
4.00	87.75	175.68	2.26	0.07	279.20
4.50	87.75	196.35	2.42	0.08	279.20
5.00	87.75	237.43	1.22	0.09	279.20
5.50	87.75	257.97	2.43	0.10	279.20
Average Velocity			2.76	Ratio (E:X)	
Average Shapes Length		E	29.38	0.54	Original Height (mm)
		X	25.50	0.46	55.00

Fig. 6 Graphical behavior of force, stress & velocity at no lubricant



Final Data for Red Oil Lubricant					
X (mm)	$F x10^{3}$ N	T (Sec)	V(m/s)	$\mathcal{E}$=(ΔH/H)	σ x10^{6} N/m^2
0.00	0.00	0.00	0.00	0.00	0.00
0.50	23.50	14.55	3.55	0.01	74.77
1.00	35.00	31.96	2.90	0.02	111.36
1.50	43.50	50.81	2.67	0.03	138.41
2.00	54.50	71.07	2.48	0.04	173.41
2.50	61.00	92.68	2.34	0.05	194.09
3.00	63.50	111.67	2.63	0.05	202.05
3.50	66.00	135.12	2.13	0.06	210.00
4.00	65.50	153.57	2.98	0.07	208.41
4.50	65.00	171.87	2.79	0.08	206.82
5.00	65.00	191.04	2.64	0.09	206.82
5.50	65.00	219.34	1.92	0.10	206.82
Average Velocity			2.76	Ratio (E:X)	
Average Shapes Length		E	29.38	0.54	Original Height (mm)
		X	25.50	0.46	55.00

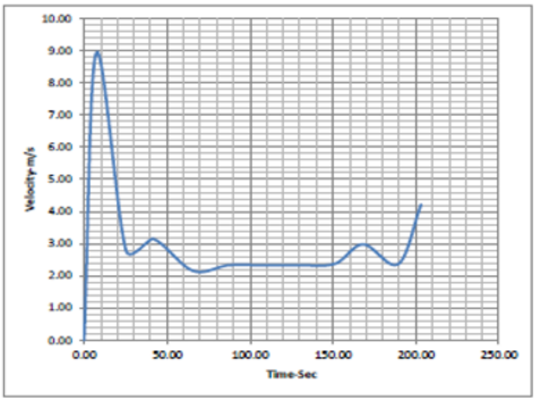
Fig. 7 Graphical behavior of force, stress & velocity using red oil as lubricant

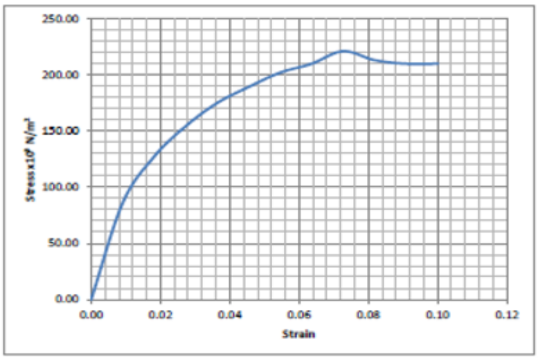


Final Data for Break Oil Lubricant					
X (mm)	$F x10^{3}$ N	T (Sec)	V(m/s)	$\mathcal{E}$=(ΔH/H)	σ x10^{6} N/m^2
0.00	0.00	0.00	0.00	0.00	0.00
0.50	20.50	4.12	12.23	0.01	65.23
1.00	36.50	18.99	3.52	0.02	116.14
1.50	46.50	38.33	2.59	0.03	147.95
2.00	54.50	57.55	2.60	0.04	173.41
2.50	60.00	78.24	2.42	0.05	190.91
3.00	63.00	98.98	2.41	0.05	200.45
3.50	65.00	124.32	1.97	0.06	206.82
4.00	64.00	144.23	2.54	0.07	203.64
4.50	62.50	161.84	2.90	0.08	198.86
5.00	62.50	179.37	2.87	0.09	198.86
5.50	62.50	205.46	1.99	0.10	198.86
Average Velocity			2.76	Ratio (E:X)	
Average Shapes Length		E	29.38	0.54	Original Height (mm)
		X	25.50	0.46	55.00

Fig. 8. Graphical behaviour of force, stress & velocity using brake oil as lubricant

Final Data for Mustard Oil Lubricant					
X (mm)	$F \times 10^3$ N	T (Sec)	V(m/s)	ε=(ΔH/H)	$\sigma \times 10^6$ N/m²
0.00	0.00	0.00	0.00	0.00	0.00
0.50	27.00	7.36	8.93	0.01	85.91
1.00	40.00	25.33	2.78	0.02	127.27
1.50	48.50	41.66	3.15	0.03	154.32
2.00	55.00	65.63	2.16	0.04	175.00
2.50	59.50	87.11	2.33	0.05	189.32
3.00	63.50	108.61	2.33	0.05	202.05
3.50	66.00	130.11	2.33	0.06	210.00
4.00	69.50	151.23	2.38	0.07	221.14
4.50	67.00	168.25	2.98	0.08	213.18
5.00	66.00	189.38	2.37	0.09	210.00
5.50	66.00	203.10	4.22	0.10	210.00
Average Velocity			2.76	Ratio (E:X)	
		E	29.38	0.54	Original Height (mm)
Average Shapes Length		X	25.50	0.46	55.00





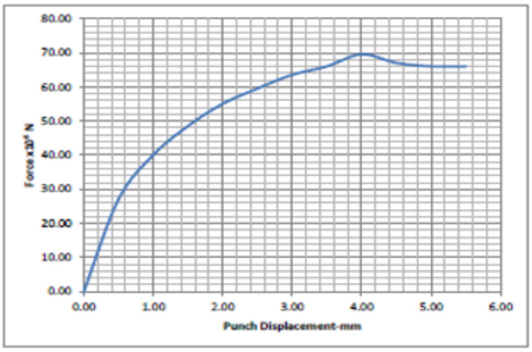


Fig. 9. Graphical *behaviour* of Force, Stress & Velocity using mustard oil as lubricant

The results obtained upon the application of the upper bound theorem in this piece of work explain that the analytical plasticity methods lead to an accurate estimation of key parameters in complex extrusion flows. Near agreement of the measured over calculated extrusion loads, pressure, strain rates and effective strain values is a good sign that the upper bound method can be an easy and efficient way to estimate the process and tool design in the metal forming process (Wan & Ren, 2015). This discussion presents a critical questioning of these findings with regard to the assumptions under which upper-bound modelling works by comparing them with other findings in the available literature regarding the topic in academia. The upper bound model exhibition of forming loads indicated a variation spreading between 3.7% to 6.7% of experimental results. Such high levels of correlation reinforce the assertion by (Ayer, 2017) and Hajiahmadi *et al.* (2024) that upper-bound solutions, when constructed with physically realistic and kinematically admissible velocity fields, can provide upper-limit estimates that are highly practical for engineering applications. The fact that the load required at the higher viscosity of lubricant shows a trend that is very consistent with the model known to be sensitive to interfacial friction parameters, which has been long established in extrusion literature (He *et al.*, 2023). Achievement of such accuracy was made possible by the use of realistic friction factors in the model, which were calibrated by means of experimental observations. Among the advantages of the upper bound method is the fact that it models the energy dissipation parts of deformation, which consist of the internal deformation work, interface frictional work and shear banding. These components were taken into consideration during the formulation of the model, and this was evident in its predictions. The higher deviation experienced in the testing of mustard oil (6.7%) compared to brake oil (2.9%) can probably be attributed to the variable surface coverage of the former while measuring friction, which is also a well-known weakness of experimental frictional analysis (Vashishtha *et al.*, 2025). The values of the model in identifying the extrusion forces and pressures lie within 7 per cent of the experimental results, showing the effectiveness of the approach in undertaking a relatively rough analysis of a metal forming process (Leśniak *et al.*, 2024).

Besides, the beautiful, good, effective strain, irrespective of testing, dilute strain rate prediction, indicates that a model has great prospects to estimate geometric and kinematic qualities of deformation (Durdag *et al.,* 2018; Pragana *et al.,* 2024). These finding backs up the assertion claimed by Duarte *et al.* (2020) that tribological conditions in metal forming are more often than not complex, requiring a hybrid experimental-analytical treatment to achieve a complete characterisation. These results of the extrusion pressure also correlate with the theoretically anticipated plastic deformation by the confined die flowing plastic. The slight overestimation of pressure values in the upper-bound model can be attributed to its foundational assumption of ideal plasticity and neglect of elastic deformation, which, while acceptable in most forming scenarios, can lead to conservative estimates (Rossi *et al.,* 2022). Nonetheless, this conservatism is often beneficial in early-stage process design where safety margins are valued. Furthermore, the consistency in pressure trends between model and experiment validates the effectiveness of integrating shape-dependent extrusion area ratios into the model's formulations. The ability of the model to predict effective strain and strain rate also gives credence to the model. The strain, computed by dividing the natural logarithm of the extrusion ratio, was not allowed to vary among tests, but it was logical since the geometries of the billet and die were similar. However, there was a significant difference in the strain rate on the basis of the speed of rams and the complexity of flow paths encountered. The average strain rate suggested by the model is 16.54 s-1, which is close to that reported by Li *et al.* (2022) For the same extrusion shapes. Importantly, the FEM simulations validated these estimates, revealing intense deformation zones near the die entry points and confirming the presence of the velocity discontinuities assumed in the analytical derivations. Despite the model's strengths, some limitations must be acknowledged. The accuracy of upper bound results depends heavily on the appropriateness of the selected velocity field. In this study, the assumed radial-axial velocity profile was simplistic yet adequate for the 'E' and 'X' profiles. However, for more asymmetric or multi-step geometries, the velocity field construction becomes significantly more complex, potentially diminishing the model's efficiency and accuracy (Ren *et al.,* 2021). Additionally, temperature effects were neglected in this cold extrusion model, but in industrial warm or hot extrusion scenarios, thermal softening and temperature-dependent friction would need to be accounted for, as discussed by (Waanders *et al.,* 2020).

One of the main implications of these results is that the upper bound theorem is useful in the optimisation of process parameters. Pre-analysis results on the possibility to manipulate the friction factors, flow path assumptions, and die geometry before the physical testing can save on cost as well as lead time. Furthermore, coupled with finite element analysis, as in this present research, upper bound models can be used to verify and act as a tool of sensitivity, giving bound estimates to envelope numerical model results and unearth unrealistic model simulation results (Wan & Ren, 2015). To sum up, the study has justified the topicality and usefulness of the upper bound theorem to forecast the mechanical and energy demands of complicated extrusion procedures. It is used in procedures that include the empirical calibration and numerical check of its predictive power. Upper bound modelling is seen as a useful tool in the initial stages of process planning, providing good quality information and predictions in a relatively short period of time, which can be used to guide process design of tools, materials, and lubricants.

CONCLUSION

The upper bound theorem has been an assessment to determine the feasibility of predicting how much load, pressure, strain, and strain rate of the cold extrusion of lead billets will be generated by using complex-shaped dies. The similarity between values offered by the theoretical estimations, experiment and finite element analysis shows that the theory used provides similar values and can be employed with carefully selected velocity fields and calibrated frictional conditions. The fact that the upper-bound model is sensitive to input effects in the lubrication process, in that it affects forming load and energy dissipation, was a justification for the use of the model during comparative analysis and process optimisation.

Although the modelling of complex asymmetric flows or effects on the thermal condition still tends to remain on the conservative side, the integration of upper bound solutions with empirical measurements and FEM has provided a strong basis that the metal forming process can be validated against. Such methodology introduces a cost-effective and feasible approach to early design and optimisation of the extrusion process in both research and industry environments.

CONFLICT OF INTEREST

There is no conflict of interest in this research review.

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